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A GENERAL METHOD OF CONSTRUCTING THREE-DIMENSIONAL WEIGHT FUNCTIONS FOR ELASTIC BODIES WITH CRACKS†

A. N. BORODACHEV

Kiev

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A new general method of constructing weight functions (WFs) for three-dimensional elastic bodies with planar cracks is presented. It is shown that in order to obtain the WFs it suffices to know a sample solution of the problem under consideration corresponding to the simplest load on the crack surfaces as well as Green's function for the domain occupied by the crack. As an example a three-dimensional WF is constructed for a strip-shaped crack in an unbounded homogeneous isotropic elastic body.

1. WEIGHT FUNCTIONS AND VARIATIONAL FORMULA

IN A CARTESIAN system of coordinates x_1, x_2, x_3 we consider the static problem for an elastic body V bounded by a surface $\Omega=\Omega_1\cup\Omega_2$, where Ω_1 and Ω_2 have no common interior points. Homogeneous kinematic boundary conditions are given on Ω_1, while homogeneous static boundary conditions are given on Ω_2. For an unbounded body V the boundary conditions on Ω can be reduced to the standard decay conditions for an elastic field at infinity. The body contains an inner crack occupying a domain S in the plane $x_3=0$. The boundary Γ of S may consist of m sufficiently smooth segments, i.e. $\Gamma=\Gamma^{(1)}\cup\Gamma^{(2)}\cup\ldots\cup\Gamma^{(m)}$, where $\Gamma^{(j)}$ have no common points. Ring-shaped and strip-shaped cracks provide obvious examples of cracks whose contours consist, for example, of two segments ($m=2$). We denote the points on the surfaces of S by $Q=(x_1,\ x_2,\ 0)$ and $Q^0=(x_1^0,\ x_2^0,\ 0)$, and the points belonging to Γ by $M=(y_1,\ y_2,\ 0)$.

We shall only consider the case (the most important one from the viewpoint of applications) of a normal rupture crack whose surfaces are subjected to an arbitrary self-balanced system of normal loads $p(Q)$ when there are only tangential loads in the plane $x_3=0$. The case when the load is applied at a distance from the crack can be reduced to the case under consideration by means of the superposition principle (sometimes also called the Buckner principle) [1].

The weight function (WF) $W^{(j)}(Q^0;\ M)$ for a segment $\Gamma^{(j)}$ of the crack contour is, by definition, equal to the stress intensity factor (SIF) $K_1^{(j)}(M)$ due to the load $p(Q)=\delta(Q;\ Q^0)$, where δ is the two-dimensional Dirac function. For an arbitrary load $p(Q)$ on the crack surface, the SIF can be expressed in terms of the corresponding WF with the aid of quadratures

$$K_1^{(j)}(M)=\iint\limits_S W^{(j)}(Q;M)p(Q)dS(Q) \tag{1.1}$$

where dS is an element of area in Cartesian coordinates.

We assume that a (sample) solution of the problem under consideration for a body V with a crack is known, which satisfies the boundary conditions on the surface Ω of the body and

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corresponds to a load $p(Q)$ on the crack surface, i.e. the functions $u_3(Q)$ and $K_1^{(j)}(M)$ are given for this sample solution, $u_3(Q)$ being the normal displacements of the points on the crack surfaces. In this case, the variation of the normal displacements of the points on the surfaces of the crack due to a small variation $\delta\Gamma$ of the crack contour is defined by the variational formula [2–4]

$$\delta u_3(Q) = C\int_{\Gamma} W(Q;M)K_1(M)\delta\Gamma(M)ds(M) \tag{1.2}$$

where ds is the arc element on Γ and $\delta\Gamma$ is measured in the direction of the outward normal to Γ.

The constant C in (1.2) depends only on the elastic properties of the material and can be computed from the principal terms of the asymptotic expansions of normal loads and displacements in the neighbourhood of the crack contour [4]. Thus, if

$$\sigma_{33} = K_1(2\rho)^{-1/2}, \quad u_3 = cK_1(2\rho)^{1/2}, \quad \rho \to 0$$

where ρ is the distance measured from the contour of the crack in the plane $x_3 = 0$ and c is a constant depending on the elastic properties of the material, then $C = \pi c/2$. For example, for an isotropic homogeneous elastic material the constant has the form $C = \pi(1-\nu)/(2\mu)$, where ν is Poisson's ratio and μ is the shear modulus.

We shall choose a variation of Γ such that only the variation $\Gamma^{(j)}$, i.e. $\delta\Gamma^{(j)}$, is non-zero. Then the variational formula (1.2) takes the form

$$\delta^{(j)}u_3(Q) = C\int_{\Gamma^{(j)}} W^{(j)}(Q;M)K_1^{(j)}(M)\delta\Gamma^{(j)}(M)ds(M) \tag{1.3}$$

and defines the variation of normal displacements due to the variation of $\Gamma^{(j)}$ alone.

2. A METHOD OF CONSTRUCTING WEIGHT FUNCTIONS

In the domain S occupied by the crack we consider several Dirichlet problems for the Laplace equation with respect to $f^{(j)}(Q)$ $(j = 1, 2, \ldots, m)$

$$\Delta f^{(j)}(Q) = 0, \quad f^{(j)}(M) = f_0^{(j)}(M) \tag{2.1}$$

where Δ is the two-dimensional Laplace operator in x_1 and x_2 and the boundary values of $f^{(j)}(M)$ are defined by $f_0^{(j)}(M) = 1$ if $M \in \Gamma^{(j)}$ and $f_0^{(j)}(M) = 0$ otherwise.

We know [5] that the solutions of the Dirichlet problems (2.1) can be expressed directly in terms of Green's function

$$f^{(i)}(Q) = -\int_{\Gamma^{(i)}} \frac{\partial G(Q;M)}{\partial n^{(j)}(M)} ds(M) \tag{2.2}$$

for S, where $\partial G(Q;\, M)/\partial n^{(j)}(M)$ is the derivative of Green's function $G(Q;\, Q^0)$ in the direction of the outward normal to $\Gamma^{(j)}$.

Multiplying both sides of (1.3) by $f^{(j)}(Q)$ and taking into account that the variations $\delta^{(j)}u_3(Q)$ of normal displacements do not vanish in S (in each particular case this condition can be verified directly), we obtain the following representations of the harmonic functions from (2.1)

$$f^{(j)}(Q) = C\int_{\Gamma^{(j)}} \frac{f^{(j)}(Q)W^{(j)}(Q;M)K_1^{(j)}(M)\delta\Gamma^{(j)}(M)}{\delta^{(j)}u_3(Q)} ds(M) \tag{2.3}$$

Now, comparing the representations (2.2) and (2.3), we can establish the general structure of three-dimensional WFs, namely

$$W^{(j)}(Q;M) = -U^{(j)}(Q;M)H^{(j)}(Q;M) \tag{2.4}$$

$$U^{(j)}(Q;M) = \frac{\delta^{(j)}u_3(Q)}{CK_1^{(j)}(M)\delta\Gamma^{(j)}(M)}, \quad H^{(j)}(Q;M) = \frac{\partial G(Q;M)/\partial n^{(j)}(M)}{f^{(j)}(Q)}$$

Thus, by (2.4), it suffices to know only $U^{(j)}$ and $H^{(j)}$ in order to construct the three-dimensional WFs. Clearly, the functions $H^{(j)}$ depend only on the shape of the crack and not on the geometry of V, the boundary conditions on Ω, or the elastic properties of the material. Therefore, for the standard forms of planar cracks it suffices to compute these functions only once. The geometry of V, the boundary conditions on Ω, and the elastic properties of the material are accounted for in (2.4) through the functions $U^{(j)}$ determined from a sample solution of the problem for a body with a crack, which, naturally, depends on the aforesaid characteristics.

The problem of finding Green's function and, consequently, also $H^{(j)}$ for a simply connected crack domain S can be reduced to the search for a conformal mapping of this domain onto the unit circle. Indeed, under sufficiently general conditions, Green's function can be represented in the form [5]

$$G(Q;Q^0) = -\ln|w(z;z^0)|/(2\pi) \tag{2.5}$$

$$w(z;z^0) = [w(z) - w(z^0)]/[1 - \overline{w}(z^0)w(z)], \quad z = x_1 + ix_2, \quad z^0 = x_1^0 + ix_2^0$$

where $w(z)$ is a conformal mapping from S onto the unit circle $|w| < 1$.

For the standard crack shapes Green's function can be computed directly from (2.5) using well-known conformal transformations. The main restriction on using the general representation (2.4) for the WF is therefore the existence of the corresponding sample solutions.

3. A STRIP-SHAPED CRACK

As an example of the application of the proposed general method for obtaining three-dimensional WFs, we shall construct the WF for a strip-shaped crack in an unbounded homogeneous isotropic elastic body such that $S = \{Q: -a < x_1 < a, -\infty < x_2 < \infty, x_3 = 0\}$ and the boundary contour Γ of the crack consists of two segments $\Gamma^{\pm} = \{M: y_1 = \pm a, -\infty < y_2 < \infty, y_3 = 0\}$. Here and in what follows the index j in $\Gamma^{(j)}$ will be replaced by a "plus" or "minus" sign, corresponding to the segments $\Gamma^{(\pm)}$ of the crack front.

By symmetry, it is clear that the WFs for $\Gamma^{(\pm)}$ are related by $W^{(-)}(x_1, x_2; y_2) = W^{(+)}(-x_1, x_2; y_2)$, which enables us to confine ourselves to computing, for example, only the WF $W^{(+)}$.

In the case of a strip-shaped crack domain Green's function occurring in (2.4) can be obtained by means of a conformal mapping. We know [5] that $w(z) = \mathrm{tg}(\alpha z/a)$, where $\alpha = \pi/(2a)$ is a conformal mapping of S onto the unit circle. After substituting this function into (2.5) and some reduction we arrive at the following representation for the desired Green's function

$$G(Q;Q^0) = -\frac{1}{4\pi}\ln\frac{\operatorname{ch}\alpha(x_2 - x_2^0) - \cos\alpha(x_1 - x_1^0)}{\operatorname{ch}\alpha(x_2 - x_2^0) + \cos\alpha(x_1 + x_1^0)} \tag{3.1}$$

Differentiating (3.1) with respect to x_1^0 and letting $x_1^0 \to a$ in the resulting expression (moreover, $x_2^0 \to y_2$), we find the normal derivative of Green's function at any point of $\Gamma^{(+)}$

$$\frac{\partial G(Q;M)}{\partial n^{(+)}(M)} = -\frac{F^{(-)}(Q;M)}{4a}, \quad F^{(\pm)}(Q;M) = \frac{\cos\alpha x_1}{\operatorname{ch}\alpha(x_2 - y_2) \pm \sin\alpha x_1} \tag{3.2}$$

Since $f^{(+)} = (x_1 + a)/(2a)$ is seen to be the solution of the corresponding Dirichlet problem (2.1), taking (3.2) into account we can write the function $H^{(+)}$ from (2.4) in the form

$$H^{(+)}(Q;M) = -\frac{F^{(-)}(Q;M)}{2(x_1 + a)} \tag{3.3}$$

Now, by (2.4), to construct the three-dimensional WF $W^{(+)}$ one must find $U^{(+)}$ using some known (sample) solution of the problem under consideration. When the load applied to the surfaces of the crack is independent of x_2, i.e. $p(Q) = p(x_1)$, the original three-dimensional problem involving a strip-shaped crack can be reduced to the classical problem of planar deformation for a Griffiths crack. The construction of the simplest solution for a Griffiths crack corresponds to a constant load $p(x_1) = p \equiv \text{const}$ acting on the crack surfaces and has the form [6]

$$u_3(Q) = p\mu^{-1}(1-\nu)(a^2 - x_1^2)^{1/2}, \quad K_1 = pa^{1/2} \tag{3.4}$$

Next we choose a variation of the crack contour Γ such that $\delta\Gamma = \delta\Gamma^{(+)}(M) = \delta a = \text{const}$, i.e. the original crack is subject to a small constant increment δa at every point of $\Gamma^{(+)}$. To compute the corresponding variation of normal displacements of the points on the crack surface it proves convenient to change to the new variables $x_1^0 = x_1 + d$ and $x_2^0 = x_2$, where d is a constant. The coordinates of $\Gamma^{(\pm)}$ are then described by $x_1^0 = a^{(\pm)} \equiv \pm a$, and the displacements of the crack surfaces for the sample solution (3.4) have the form

$$u_3(Q^0) = p\mu^{-1}(1-\nu)(x_1^0 - a^{(-)})^{1/2}(a^{(+)} - x_1^0)^{1/2} \tag{3.5}$$

The normal displacement variation (3.5) due to the above-mentioned variation of the crack contour can be computed as follows:

$$\delta^{(+)}u_3(Q^0) \equiv \frac{\partial u_3(Q^0)}{\partial a^{(+)}}\delta a = \frac{p(1-\nu)(x_1^0 - a^{(-)})^{1/2}\delta a}{2\mu(a^{(+)} - x_1^0)^{1/2}}$$

which, on reverting to the original variables, leads to the following result

$$\delta^{(+)}u_3(Q) = \frac{p(1-\nu)(a + x_1)^{1/2}\delta a}{2\mu(a - x_1)^{1/2}} \tag{3.6}$$

Now, substituting (3.6) the second relation in (3.4), and also the above values of C and the variations $\delta\Gamma^{(+)}$ of the crack contour into the expression for $U^{(i)}(Q;\, M)$ in (2.4), we find that

$$U^{(+)}(Q;M) = \frac{(a + x_1)^{1/2}}{\pi a^{1/2}(a - x_1)^{1/2}} \tag{3.7}$$

To obtain $W^{(+)}$ it suffices to substitute (3.3) and (3.7) into (2.4). Using this and the relationship between $W^{(-)}$ and $W^{(+)}$, we find the following explicit representation of the three-dimensional WFs for a strip-shaped crack in an unbounded homogeneous isotropic elastic body

$$W^{(\pm)}(Q;M) = \frac{F^{(\mp)}(Q;M)}{2\pi a^{1/2}(a^2 - x_1^2)^{1/2}} \tag{3.8}$$

In accordance with the concept of a WF, the SIF of normal rupture is defined by (3.8) for the points belonging to the segments $\Gamma^{(\pm)}$ of the crack contour under consideration with coordinates y_2 when the load on the crack surfaces is represented by two concentrated normal unit forces acting in opposite directions (widening the crack) applied at the points with coordinates x_1 and x_2. Unlike the representation of the three-dimensional WFs for a strip-shaped crack proposed earlier in [7, 8], formula (3.8) provides the exact solution of the problem under consideration.

By a passage to the limit in (3.8) from the crack under consideration to a half-plane-shaped crack one can verify (the computations, which are straightforward, are omitted) that the WF from (3.8) turns into the well-known three-dimensional WF for a half-plane-shaped crack [8, 9].

As a matter of fact, the problem of determining the SIF for a strip-shaped crack in an unbounded homogeneous isotropic elastic body subject to an arbitrary self-balanced system of normal loads $p(Q) \equiv p(x_1, x_2)$ can now be reduced to computing quadratures. Indeed, substituting (3.8) into (1.1) we obtain the following result

$$K_1^{(\pm)}(y_2) = \frac{1}{2\pi a^{1/2}} \int_{-a}^{a} \int_{-\infty}^{\infty} \frac{p(x_1, x_2) F^{(\mp)}(Q; M) dx_2 dx_1}{(a^2 - x_1^2)^{1/2}} \tag{3.9}$$

In the case of a plane strain in which $p(x_1, x_2) = p(x_1)$ and the SIF is independent of y_2, we write (3.9) in the form

$$K_1^{(\pm)} = \frac{1}{2\pi a^{1/2}} \int_{-a}^{a} \frac{p(x_1) \cos \alpha x_1 dx_1}{(a^2 - x_1^2)^{1/2}} \int_{-\infty}^{\infty} \frac{dx_2}{\operatorname{ch} \alpha(x_2 - y_2) \mp \sin \alpha x_1} \tag{3.10}$$

The integrals with respect to x_2 can be computed in closed form and are equal to $2(a \pm x_1)/\cos \alpha x_1$, respectively. Taking this into account, we represent (3.10) as

$$K_1^{(\pm)} = \int_{-a}^{a} k^{(\pm)}(x_1) p(x_1) dx_1, \quad k^{(\pm)}(x_1) = \frac{(a \pm x_1)^{1/2}}{\pi a^{1/2} (a \mp x_1)^{1/2}} \tag{3.11}$$

where $k^{(\pm)}(x_1)$ is the two-dimensional WF for the Griffiths crack [8]. Thus, in the case of a plane strain Eq. (3.9) leads to the known result, as expected.

4. RELATIONSHIP BETWEEN THREE-DIMENSIONAL AND TWO-DIMENSIONAL WEIGHT FUNCTIONS

According to the general scheme of the method proposed, to construct three-dimensional WFs one must know, in particular, the SIF and the normal displacements of the points on the crack surfaces corresponding to a sample solution. Unfortunately, only the relationships for the SIF are given in handbooks (for example, [8, 10]), while the representations for the displacements of the points on the surfaces of the crack are left out. Thus, whenever possible, it may turn out to be preferable to use a slight modification of the method of constructing WFs, in which one does not need to know the displacements of the points on the crack surfaces, and which rests on the relationships between three- and two-dimensional WFs.

It has been proved [11] that the three-dimensional WF for a disc-shaped crack can be computed directly if the corresponding two-dimensional axisymmetric WF is known. For a strip-shaped crack a similar result holds, according to which to compute the desired three-dimensional WF it suffices to know just the corresponding two-dimensional WF for the case of a plane strain.

The two-dimensional WFs for the case of plane strain, which will be denoted by $k^{(\pm)}(x_1^0)$, correspond to the load $p(Q) = \delta(x_1 - x_1^0)$ acting on the surfaces of a strip-shaped crack, δ being the one-dimensional Dirac function. Substituting this load into (1.1), we obtain the following formula, which is written in the coordinate form

$$k^{(\pm)}(x_1) = \int_{-\infty}^{\infty} W^{(\pm)}(x_1, x_2; y_2) dx_2, \quad -a < x_1 < a \tag{4.1}$$

By symmetry, it is clear that for the class of normal rupture problems considered with no tangential stresses in the crack plane, the three-dimensional WFs for a strip-shaped crack have the following properties

$$W^{(\pm)}(x_1,x_2;y_2)=W^{(\pm)}(x_1,x_2-y_2)=W^{(\pm)}(x_1,y_2-x_2)$$

which enables us to represent (4.1) in the form

$$k^{(\pm)}(x_1)=\int_{\Gamma^{(\pm)}} W^{(\pm)}(x_1,x_2-y_2)dy_2, \quad -a<x_1<a \tag{4.2}$$

Multiplying both sides of (4.2) by the solutions $f^{(\pm)}(x_1)$ of the corresponding Dirichlet problems (2.1) for a strip and taking into account that $k^{(\pm)}(x_1)$ does not vanish for $-a\leqslant x_1\leqslant a$, by analogy with (2.3) we get

$$f^{(\pm)}(x_1)=\int_{\Gamma^{(\pm)}} \frac{f^{(\pm)}(x_1)W^{(\pm)}(x_1,x_2-y_2)}{k^{(\pm)}(x_1)}dy_2 \tag{4.3}$$

Now, comparing (2.2) and (4.3), we can establish a relationship between the three-dimensional WFs for a strip-shaped crack and the corresponding two-dimensional WFs

$$W^{(\pm)}(x_1,x_2-y_2)=-k^{(\pm)}(x_1)H^{(\pm)}(Q;M) \tag{4.4}$$

the functions $H^{(\pm)}$ being defined by (2.4).

In the same way as in the case of the representation (3.3) for $H^{(+)}$, one can also compute $H^{(-)}$, which enables us to write (4.4) in the final form

$$W^{(\pm)}(Q;M)=\frac{k^{(\pm)}(x_1)F^{(\mp)}(Q;M)}{2(a\pm x_1)} \tag{4.5}$$

which lends itself well to practical use.

To construct the three-dimensional WFs for a strip-shaped crack it is therefore sufficient to know only the corresponding two-dimensional WFs corresponding to the case of plane strain. As an example let us once more consider the problem of a strip-shaped crack in an unbounded homogeneous isotropic elastic body. Substituting the two-dimensional WFs from (3.11) corresponding to the problem in hand into (4.5), we arrive at once at the desired result given by (3.8).

Formula (4.5) can also be used to construct approximations of three-dimensional WFs if only approximate representations of the corresponding two-dimensional WFs are known. In this case one can easily verify that the maximum relative error of computing the three-dimensional WFs is equal to that for the two-dimensional WFs.

The following formula is the result of comparing the representations (2.4) and (4.4) of three-dimensional WFs

$$k^{(\pm)}(x_1)=\frac{\delta^{(\pm)}u_3(Q)}{CK_1^{(\pm)}(M)\delta\Gamma^{(\pm)}(M)}$$

This formula enables us to compute the two-dimensional WFs on the basis of two-dimensional as well as three-dimensional sample solutions of the corresponding problem of a strip-shaped crack. It can be regarded as a generalization of the well-known Rice formula [12] based exclusively on two-dimensional sample solutions.

5. AN EXAMPLE OF COMPUTING THE STRESS INTENSITY FACTORS FOR A STRIP-SHAPED CRACK

Consider the three-dimensional problem for a strip-shaped crack in an unbounded homogeneous isotropic elastic body subject to a constant normal load p given in the domain $-a < x_1 < a$, $-b < x_2 < b$, where $b > 0$ is a constant. To compute the SIF we make direct use of formula (3.9), which takes the form

$$K_1^{(\pm)}(y_2) = \frac{p}{2\pi a^{1/2}} \int_{-a}^{a} \frac{\cos\alpha x_1 dx_1}{(a^2 - x_1^2)^{1/2}} \int_{-b}^{b} \frac{dx_2}{\operatorname{ch}\alpha(x_2 - y_2) \mp \sin\alpha x_1} \tag{5.1}$$

under the given loading conditions.

By evaluating the integrals with respect to x_2 in closed form and carrying out simple transformations, Eq. (5.1) can be represented in the following final form, which contains only single quadratures

$$K_1^{(\pm)}(y_2) = K_1^{\infty} D(h, y), \quad D(h, y) = \frac{2}{\pi^2} \int_{-1}^{1} \frac{\psi(x) dx}{(1 - x^2)^{1/2}} \tag{5.2}$$

$$\psi(x) = \psi^+ - \psi^-, \quad \psi^{\pm}(x) = \operatorname{arctg} \frac{e^{\pm\pi(h \mp y)/2} + \sin(\pi x / 2)}{\cos(\pi x / 2)}, \quad h = \frac{b}{a}, \quad y = \frac{y_2}{a}$$

where $K_1^{\infty} = pa^{1/2}$ is the SIF for plane strain, which occurs in the problem under consideration when $b \to \infty$. It can be verified that $D(h, y) \to 1$ as $b \to \infty$ and (5.2) can be reduced to the well-known result for the case of a plane strain.

Quadrature formulae must be used to find $D(x, y)$ for finite values of b. The simplest way of evaluating the integral in (5.2) is by a Gauss-type integral formula with the root singularities at the ends of the integration interval appearing explicitly [13].

The function D and, consequently, also the SIF in (5.2) depend on only two dimensionless parameters h and y. The dependence of D on y is presented in Fig. 1 for some $h \leq 1$ (the solid curves) and $h > 1$ (the dashed lines). In particular, the results presented indicate that the relation $K_1^{(\pm)}(0) \approx K_1^{\infty}$ starts to hold from $h \approx 4$.

Note that the method proposed enables the class of three-dimensional problems of crack theory for which analytic (exact or approximate) solutions can be obtained to be extended considerably.

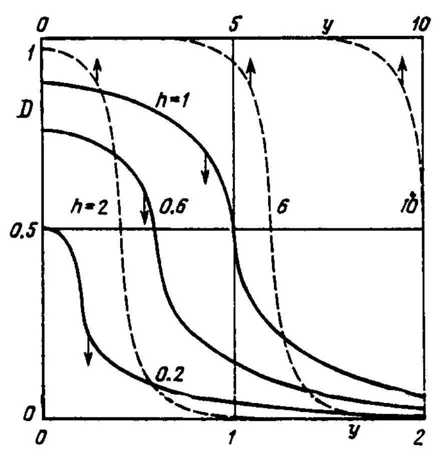

FIG. 1.

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